YiaMath

“Its all in your head”

Complex numbers

By:
Timur Bek KÖROĞLU
Kaan Mahir AY
Hayriye Sevinç ERDEM
Ege ÇİLEK

“YÜKSEL-İLHAN ALANYALI FEN LİSESİ MÜDÜRLÜĞÜ ADINA İMTİYAZ SAHİBİ Muammer OKUMUŞ”

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Timur Bek KÖROĞLU

YAYIN KURULU

Timur Bek KÖROĞLU, Kaan Mahir AY, Hayriye Sevinç ERDEM, Ege ÇİLEK

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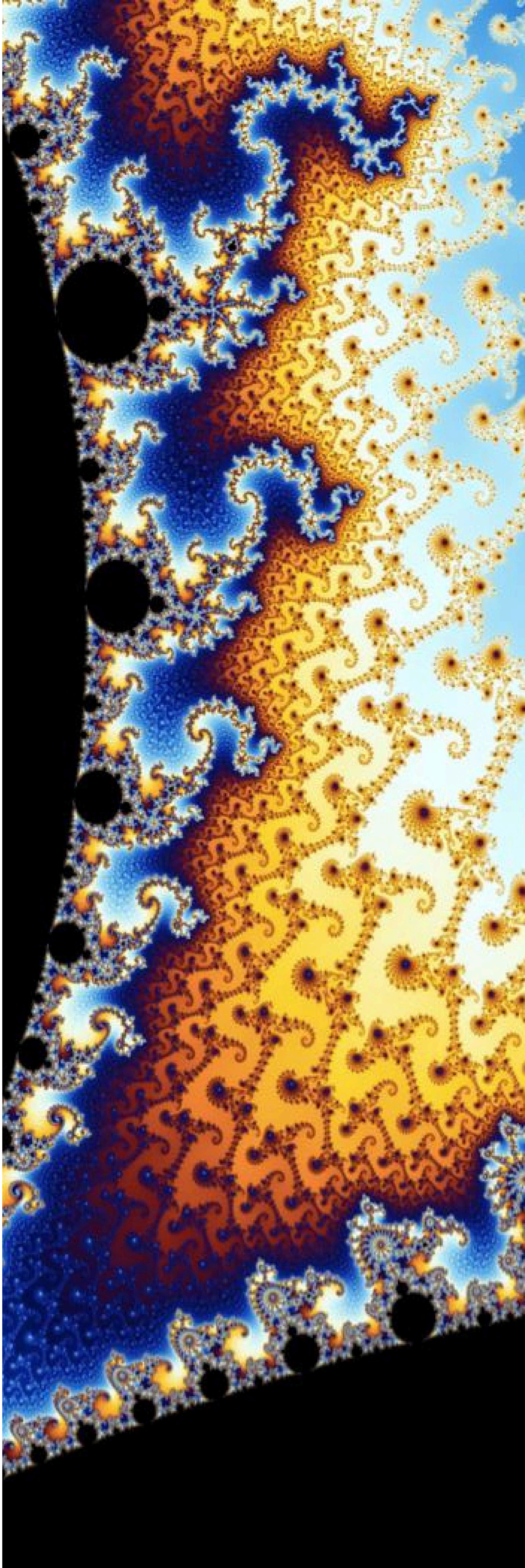
GRAFİK-TASARIM

Timur Bek KÖROĞLU

İLETİŞİM

Karlıktepe, Güneş Sk. No:1, 34870 Kartal/İstanbul, (216) 353 46 47, kartalyia34@hotmail.com



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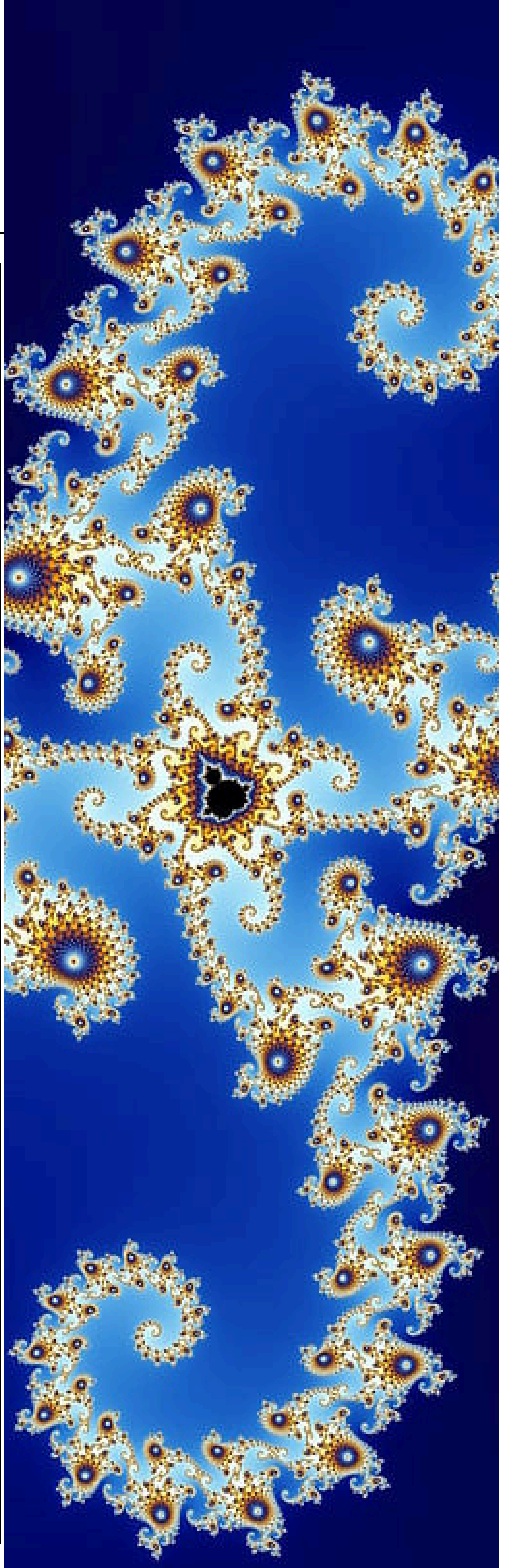
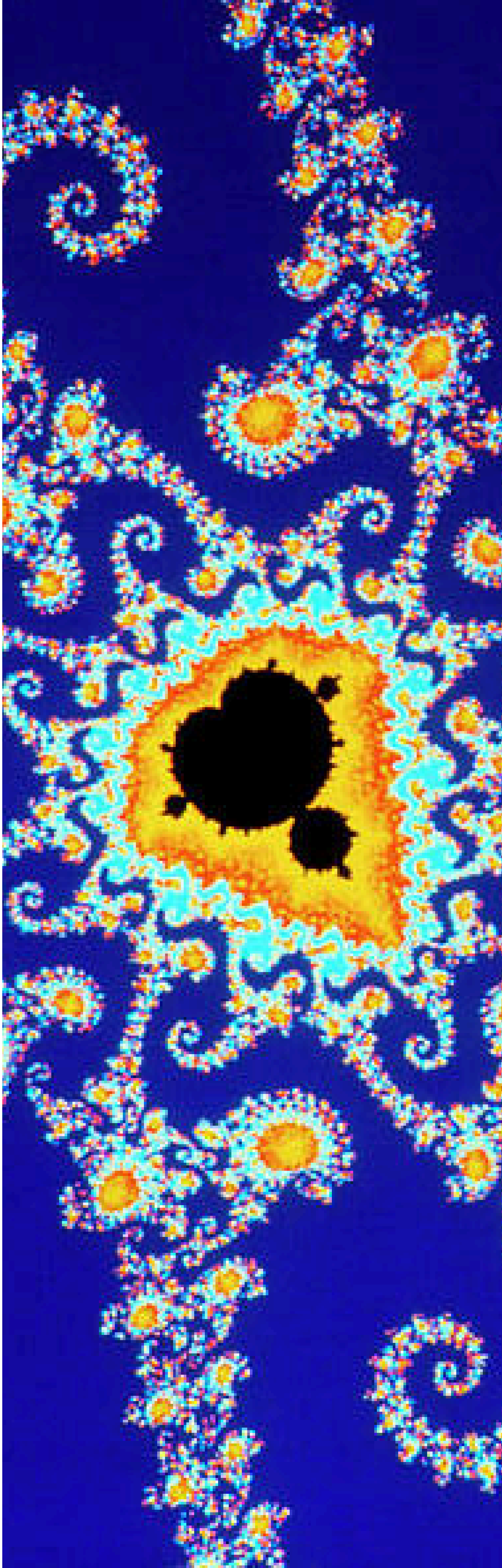


What are the complex numbers?

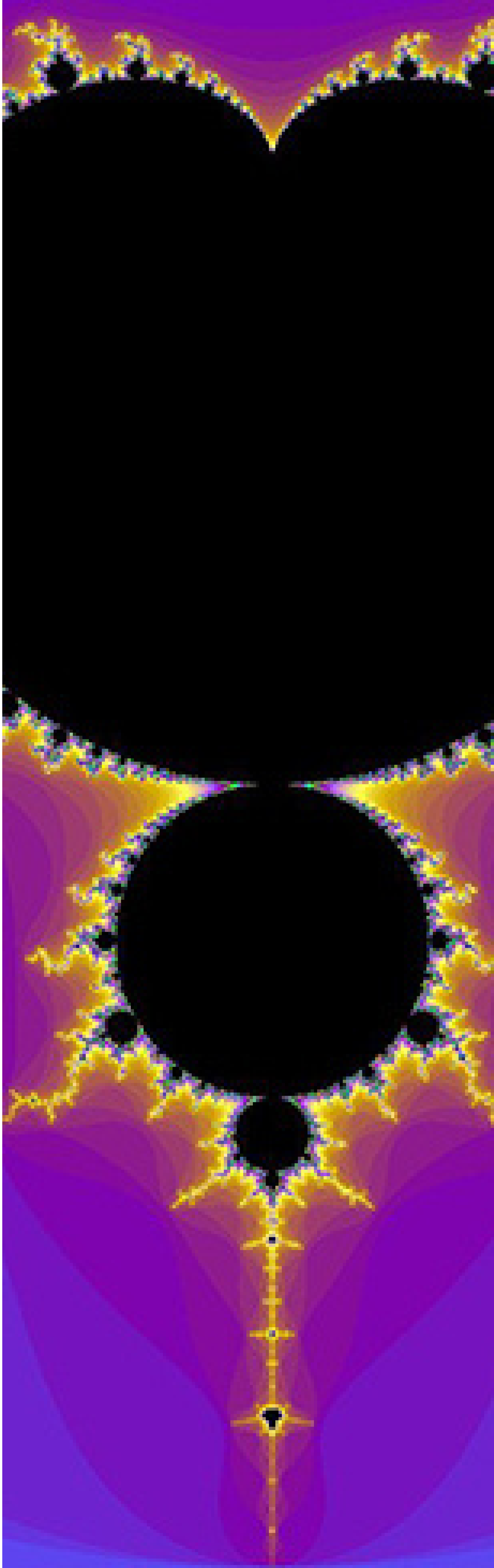
In mathematics, a complex number is an element of a number system that extends the real numbers with a specific element denoted *i*, called the imaginary unit and satisfying the equation *i*2=−1; every complex number can be expressed in the form *a*+*bi*, where *a* and *b* are real numbers. Because no real number satisfies the above equation, *i* was called an imaginary number by René Descartes.

Usage Of Complex Numbers

Complex numbers can be used in mathematics to solve problems involving the roots of negative numbers, complex derivatives and integrals, and to solve various mathematical equations including algebra and calculus, including differential equations.



Some common applications of complex numbers include:



1- In signal processing, complex numbers are used to represent and manipulate signals with both real and imaginary components.

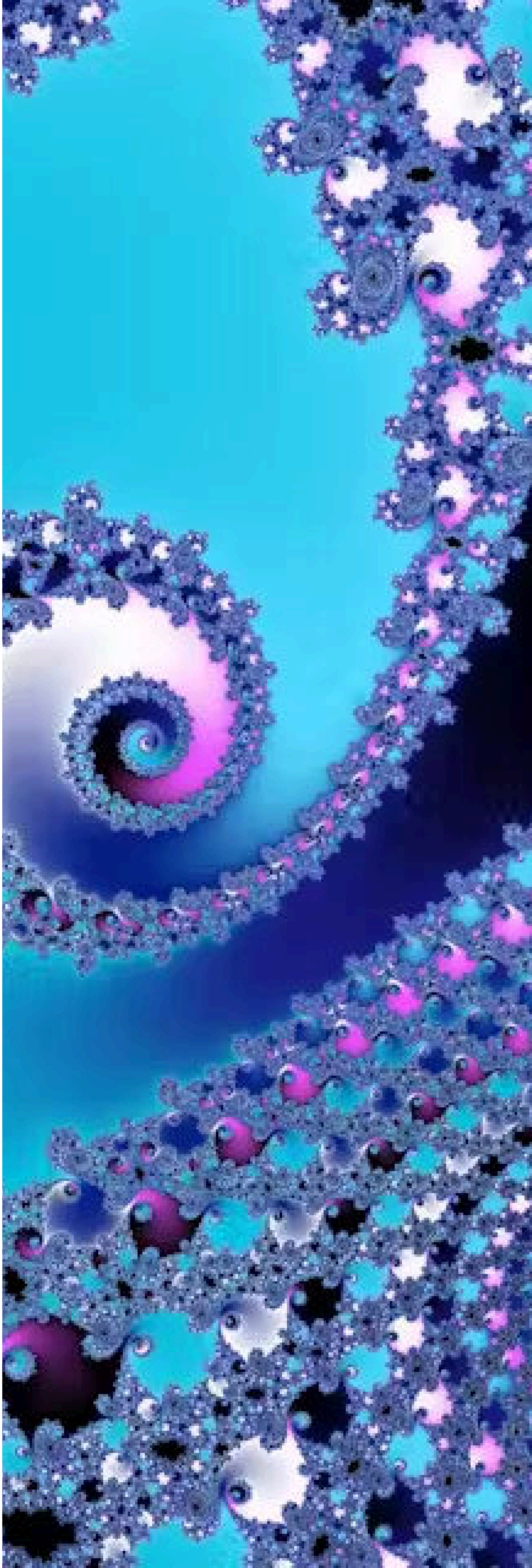
2- In quantum mechanics, physically observable quantities such as position and momentum are represented with complex numbers.

3- In electrical engineering, complex numbers are used to analyze AC circuits and calculate voltage, current, and power in circuits.

Rational numbers are used in various fields of mathematics and in the real world. Here are some examples:

"Money Transactions: Rational numbers are associated with currency units and amounts. From daily shopping to banking transactions, rational numbers are used to accurately calculate amounts of money.

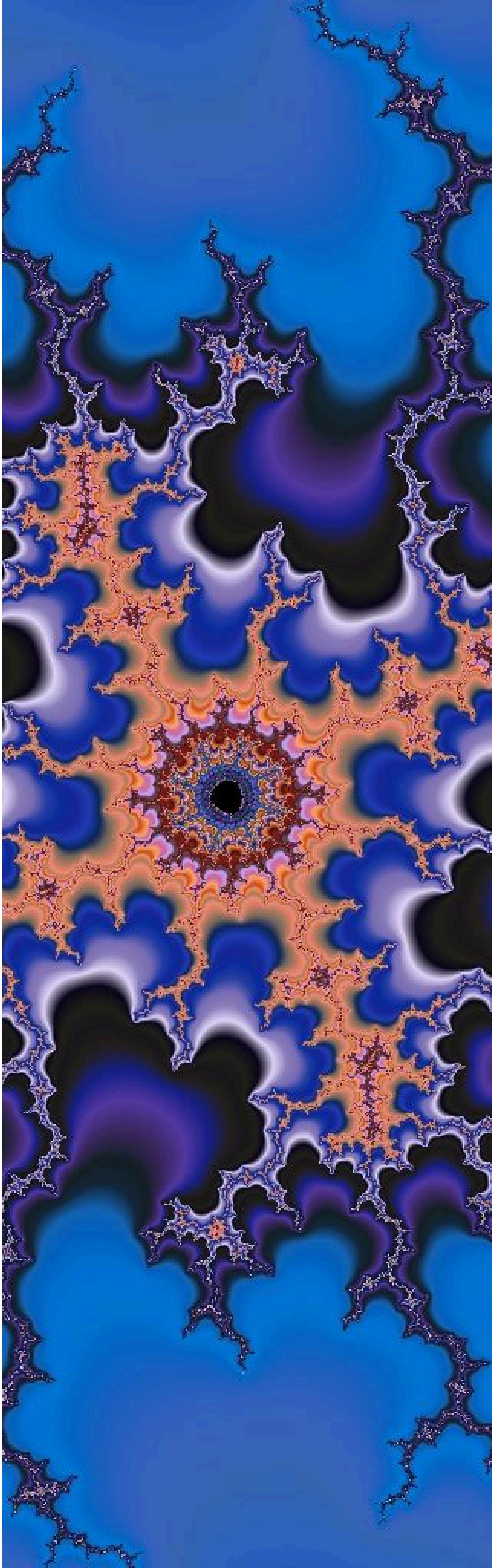
Measurement: Measurement units are generally expressed with rational numbers. Rational numbers are used in measuring physical quantities such as length, weight, and volume.



Engineering: Rational numbers are crucial in engineering fields such as construction and electrical circuit design. These numbers are used for precise calculations and accurate measurements.

Computer Science: Computer science and information technology are another area where rational numbers are widely used. In computer programming, rational numbers are often preferred for computation accuracy.

Statistics: Rational numbers frequently appear in data analysis and statistical calculations. For example, statistical concepts like mean and standard deviation are expressed with rational numbers.

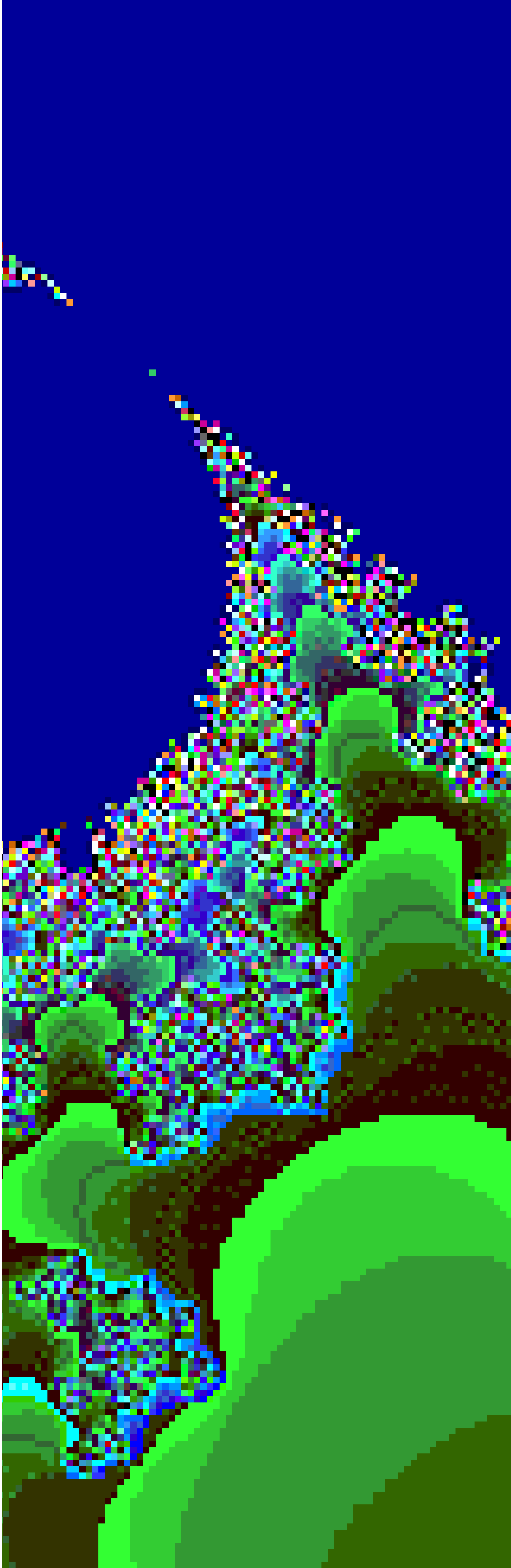


Logic and Computer Science: Rational numbers are commonly used in operations in logic circuits and digital electronics. In these areas, rational numbers are one of the fundamental building blocks enabling computers to perform operations.

The use of rational numbers in these fields is widespread and plays a significant role in many aspects of mathematics.

Operations With Complex Numbers

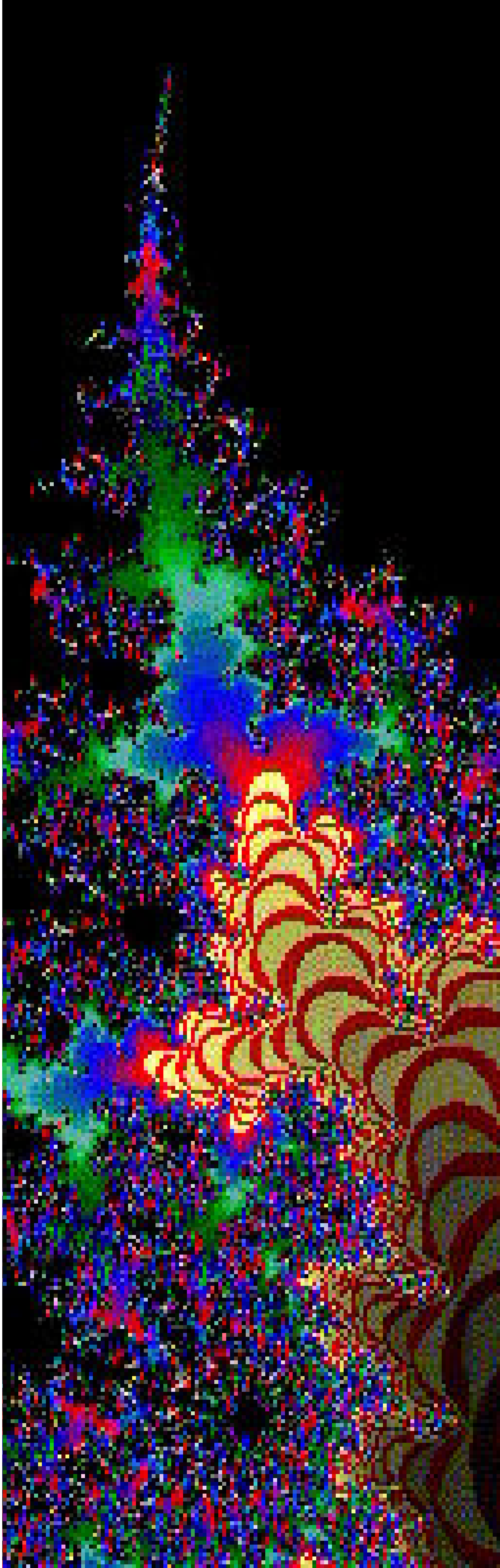
Complex numbers are numbers that consist of a real part and an imaginary part. They are written in the form a + bi, where 'a' is the real part, 'b' is the imaginary part, and 'i' is the imaginary unit, which is the square root of -1.



Here are the basic operations you can perform with complex numbers:

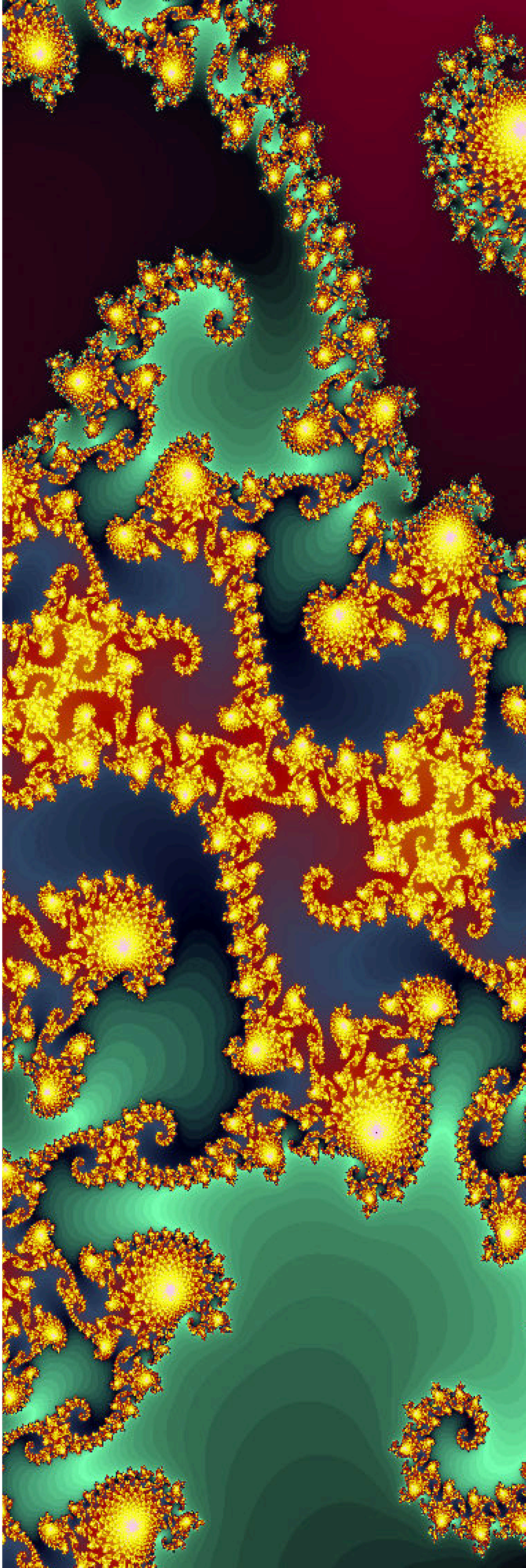
1. **Addition:** To add two complex numbers, you simply add their real parts together and their imaginary parts together. For example:
\[(a + bi) + (c + di) = (a + c) + (b + d)i \]

2. **Subtraction:** To subtract one complex number from another, you subtract their real parts and their imaginary parts separately. For example:
\[(a + bi) - (c + di) = (a - c) + (b - d)i \]



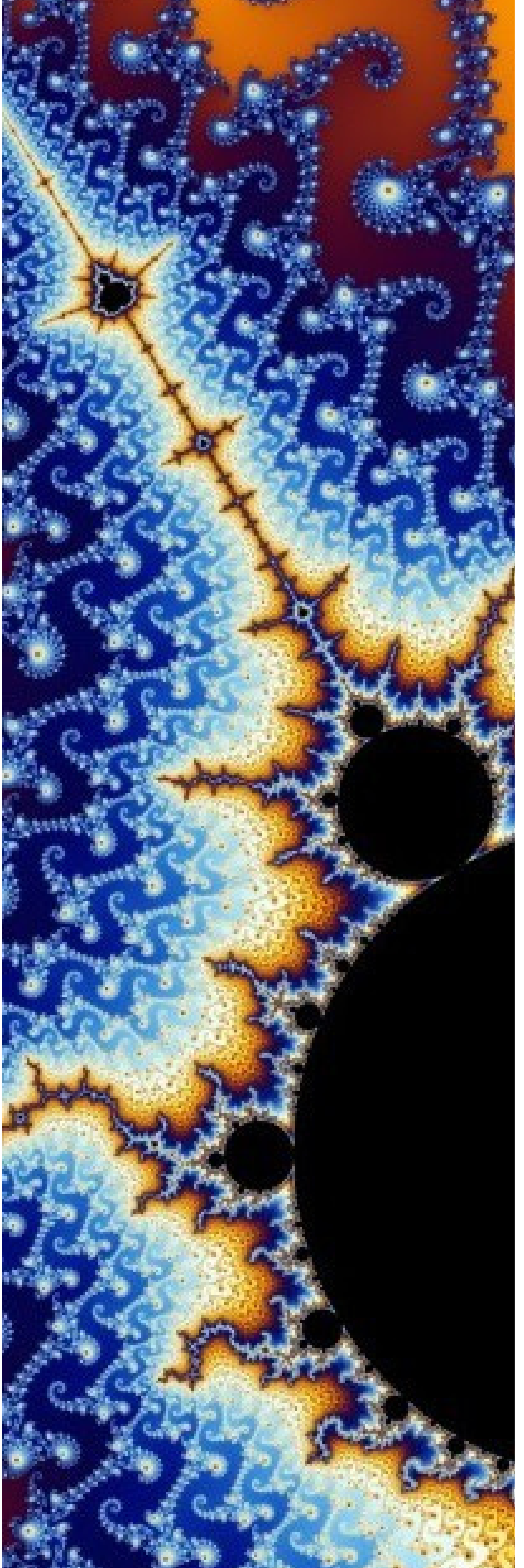
3. **Multiplication:** To multiply two complex numbers, you can use the distributive property and then simplify. For example:
\[(a + bi)(c + di) = ac + adi + bci + bdi^2 \]
Remembering that \(i^2 = -1\), you can simplify to:
\[(ac - bd) + (ad + bc)i \]

4. **Division:** To divide one complex number by another, you can use the conjugate. Multiply both the numerator and denominator by the conjugate of the denominator. The conjugate of a complex number \(a + bi\) is \(a - bi\). Then simplify. For example:
\[\frac{{a + bi}}{{c + di}} = \frac{{a + bi}}{{c + di}} \times \frac{{c - di}}{{c - di}} \]
Then simplify the numerator using FOIL and remember \(i^2 = -1\). You'll get a real part and an imaginary part.

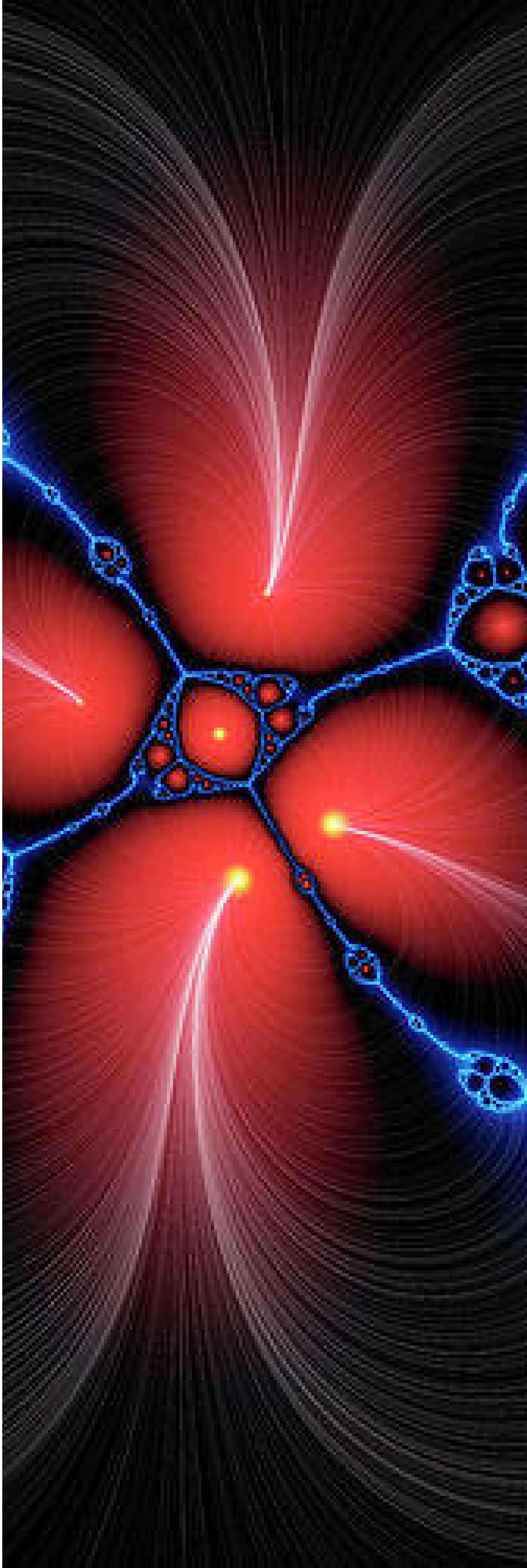


History

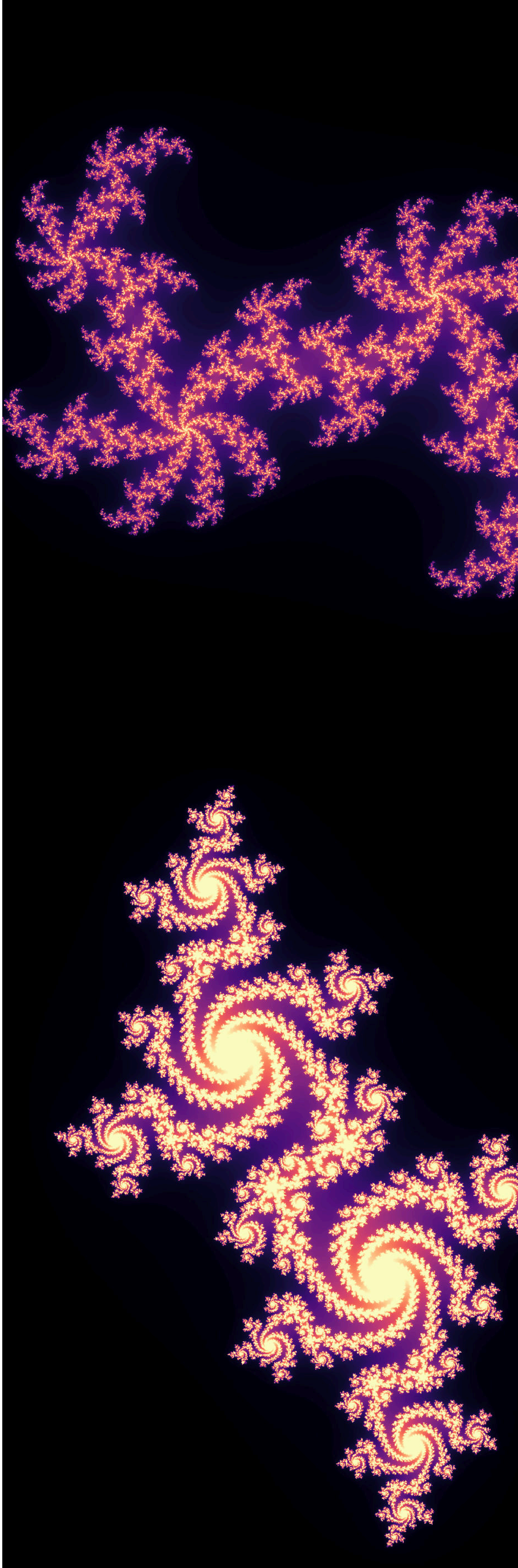
1. Origins of Complex Numbers: The roots of complex numbers extend deep into antiquity, with Babylonian mathematicians encountering equations with no real solutions around 2000 BCE. However, the formalization of these concepts awaited the Renaissance era. In the works of Italian mathematicians such as Cardano, Tartaglia, and Bombelli, the foundations of modern algebra emerged. Bombelli's treatise Algebra (1572) notably introduced the notion of imaginary numbers as a means to solve cubic equations, marking a significant step toward the understanding of complex numbers.



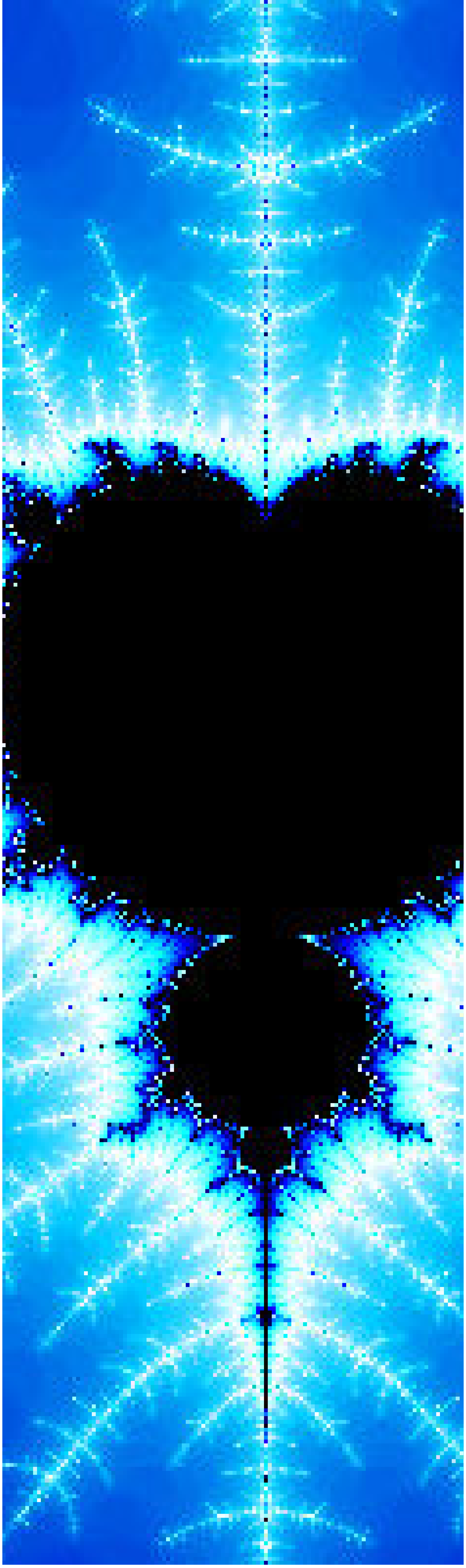
2. Renaissance and Early Modern Era: The Renaissance period witnessed a surge in mathematical exploration, spurred by advancements in commerce, science, and art. Mathematicians grappled with equations beyond the realm of real numbers, encountering challenges that would reshape mathematical thought. Notably, the solution to cubic and quartic equations presented a formidable puzzle. Scholars such as Niccolò Fontana Tartaglia and Girolamo Cardano made groundbreaking contributions, laying the groundwork for the formalization of complex numbers. However, it was Rafael Bombelli who dared to confront the notion of "imaginary" solutions in his work Algebra (1572). Bombelli's courageous approach to solving cubic equations paved the way for the eventual acceptance of complex numbers, albeit amidst lingering skepticism and controversy.



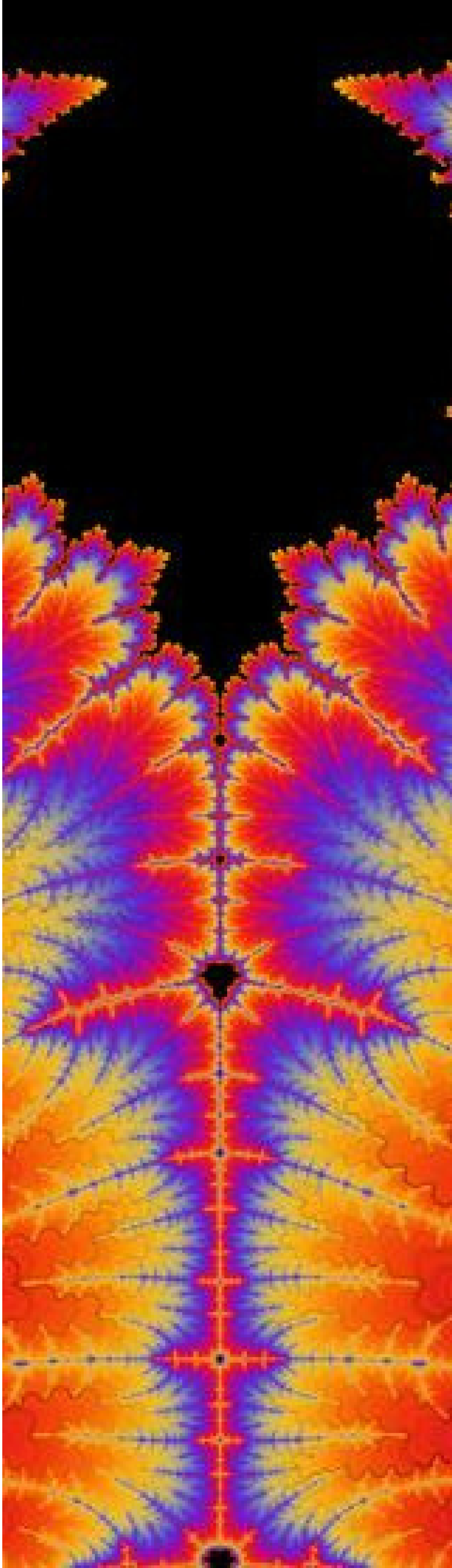
3. The Imaginary Stigma: Despite their utility in solving equations, complex numbers faced formidable opposition from mathematicians and philosophers alike. The term "imaginary" itself carried connotations of falsehood and unreality, leading to deep-seated skepticism. The renowned mathematician René Descartes, for instance, dismissed complex roots as mere artifacts of calculations rather than legitimate mathematical entities. Similarly, the philosopher George Berkeley questioned the ontological status of imaginary numbers, challenging their validity within the framework of reality. The resistance to complex numbers persisted well into the Enlightenment era, where thinkers grappled with reconciling these abstract entities with their understanding of the natural world



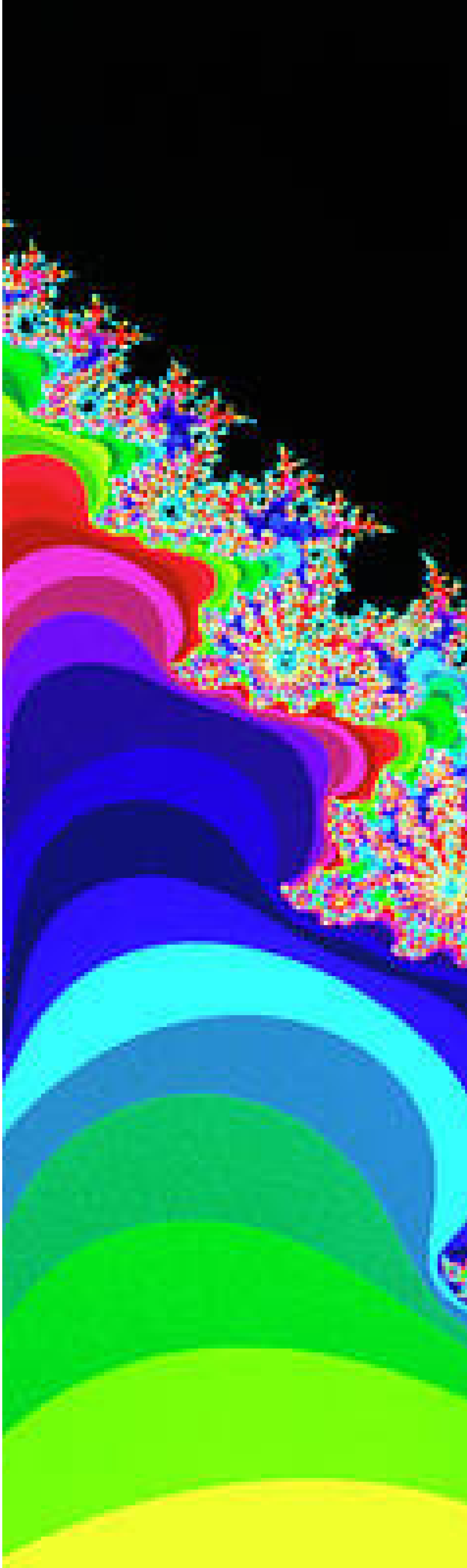
4. Euler's Contribution: The 18th century witnessed a paradigm shift in the perception of complex numbers, catalyzed by the groundbreaking work of Leonhard Euler. In his seminal treatise Introduction to the Analysis of the Infinite (1748), Euler introduced the notation “i” for the imaginary unit and elucidated the geometric interpretation of complex numbers. Euler's elegant formulations provided a unifying framework for understanding complex arithmetic, paving the way for their widespread acceptance among mathematicians. Moreover, Euler's contributions extended beyond pure mathematics, finding applications in diverse fields such as physics and engineering. Euler's bold insights not only elevated the status of complex numbers but also transformed them into indispensable tools for tackling complex problems across various disciplines.



5. Complex Analysis and Applications: The 19th century heralded a new era of mathematical rigor and exploration, with complex analysis emerging as a distinct discipline. Visionaries such as Augustin-Louis Cauchy and Carl Friedrich Gauss made pioneering contributions to the understanding of complex functions and their properties. Cauchy's foundational work on complex integration and the residue theorem laid the groundwork for modern complex analysis, establishing rigorous techniques for evaluating complex integrals and singularities. These developments not only enriched theoretical mathematics but also found practical applications in fields such as physics and engineering. Indeed, the mathematical machinery of complex analysis proved indispensable in solving a myriad of problems, from fluid dynamics to electrical circuit theory.

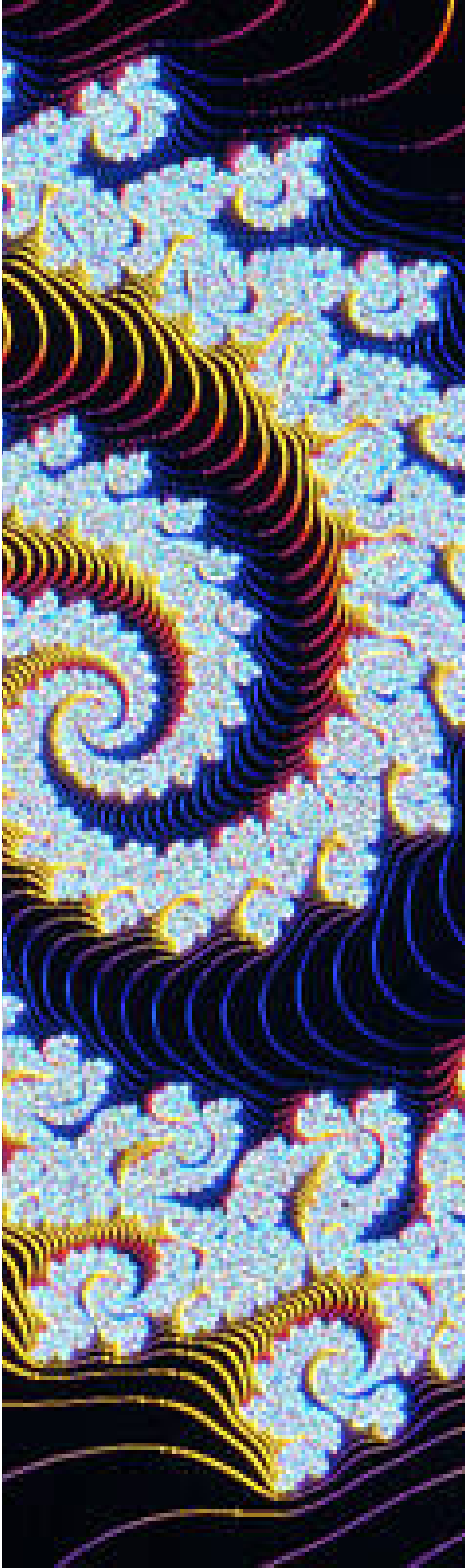


6. Geometric Interpretation and Visualizations: The geometric interpretation of complex numbers, particularly in the Argand plane, revolutionized the understanding of their properties. Introduced by Jean-Robert Argand in 1806 and later popularized by Caspar Wessel, the Argand plane provided a geometric representation of complex numbers as points in a two-dimensional plane. This visual framework enabled mathematicians to gain deeper insights into the behavior of complex functions and their geometric transformations. Furthermore, the advent of computer graphics in the 20th century allowed for the creation of intricate visualizations, such as the Mandelbrot set and Julia sets. These mesmerizing fractal patterns showcased the richness and complexity of complex dynamics,



captivating mathematicians and laypersons alike with their intricate beauty.

7. Modern Perspectives and Extensions: In the 20th and 21st centuries, complex numbers have become indispensable tools in various branches of mathematics and science. From quantum mechanics to signal processing, their applications continue to expand, driving innovation and discovery in diverse fields. Moreover, the exploration of extensions to complex numbers, such as quaternions and octonions, has opened new frontiers in mathematical research. These hypercomplex number systems, with their rich algebraic structures, offer fertile ground for exploration and applications in fields ranging from computer graphics to theoretical physics. As we venture further into the realms of abstract mathematics, the legacy of complex numbers endures, serving as a testament to the enduring power of human curiosity and imagination.



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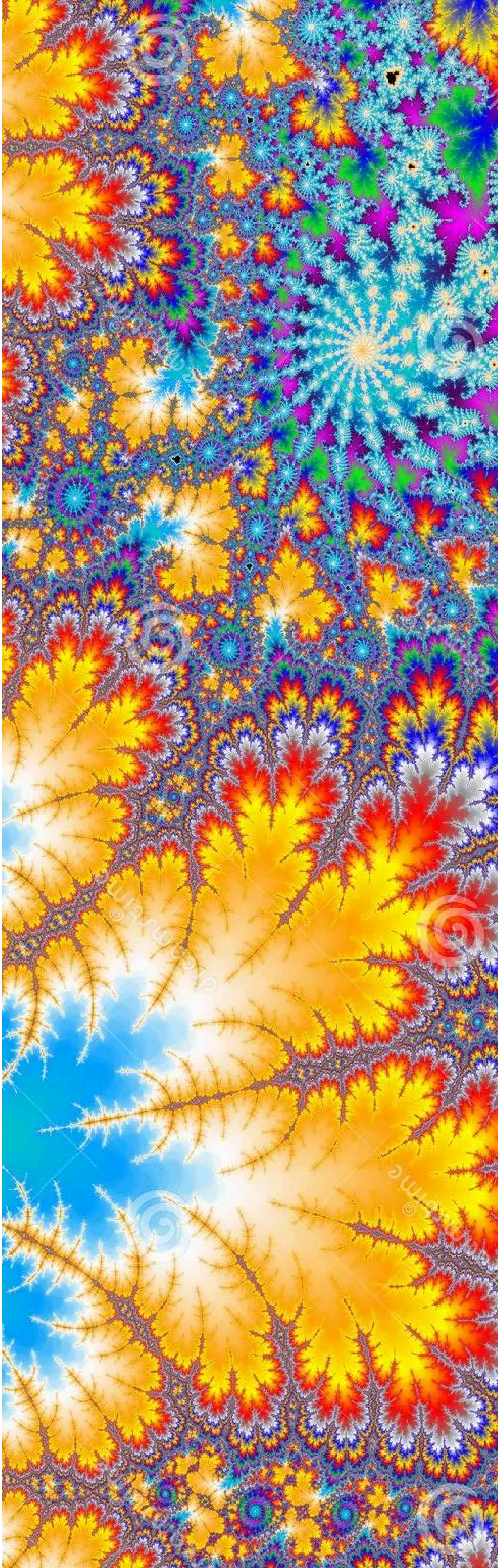


Examples to Imaginary Numbers

Some examples of imaginary numbers are -4i, 6i, i, 12.38i, -I, 3i/4, πi etc.

0 is not an imaginary number, even though we can write 0 as 0i but it is not an imaginary number as it is not associated with the square root of any negative number. so, 0 is a real number.

3i and $i \sqrt{5}$ and $-12 \sqrt{i}$ are all examples of pure imaginary numbers, or numbers of the form of bi where b is a nonzero real number.

Let's square the number 3i

$$(3i)^2 = 3^2 i^2$$

$$= 9i^2$$

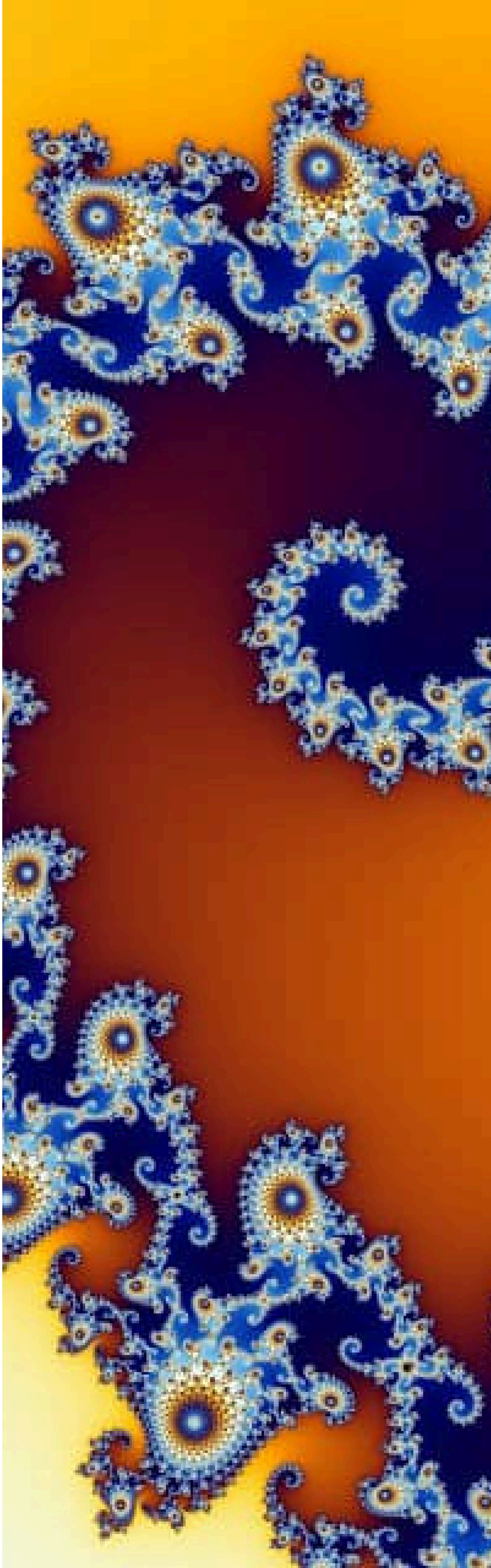
Keeping in mind that $i^2 = -1$, we can simplify it further that:

$$= 9i^2$$

$$= 9\,(-1)$$

$$= -9$$

The fact that $(3i)^2 = -9$ means that 3i is a square root of -9



Consider the unadulterated
quadratic condition:
X 2 = a, where ‘a’ could be a
known esteem. Its arrangement
may be displayed as x = va.
Hence, the rules for a few
imaginary numbers are:

i = √-1

i2 = -1

i3 = -i

i4 = +1

i4n = 1

i4n-1= -i

1 × I = i

i × I = −1

−1 × I = −i

−i × I = 1

i = √−1

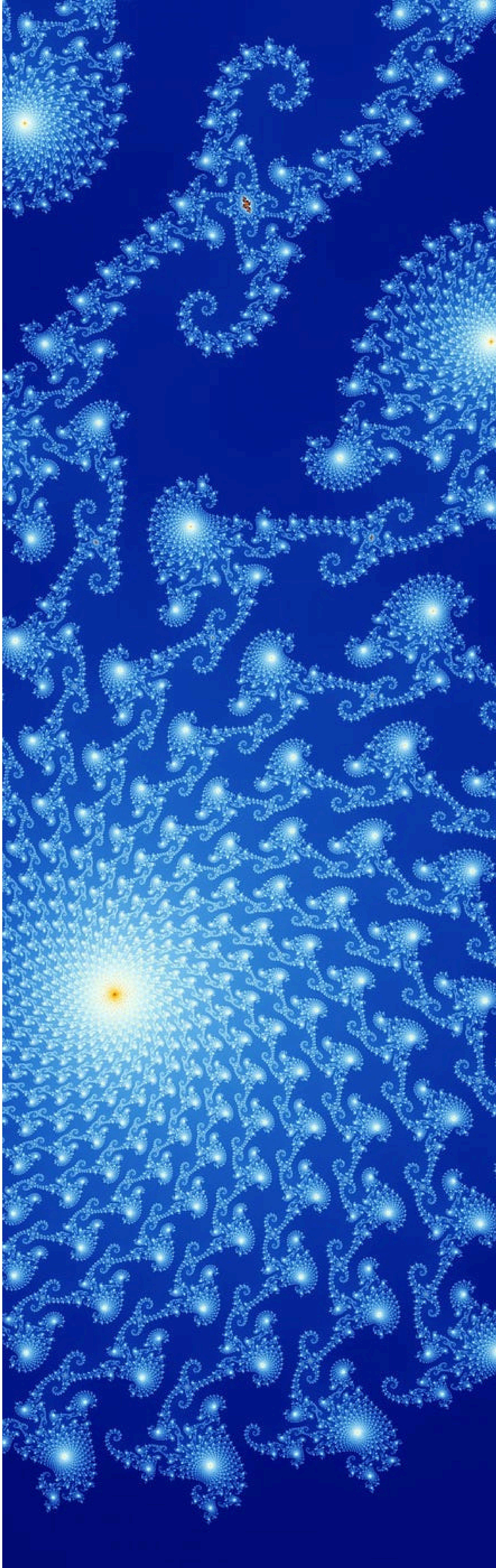
i2 = −1

i3 = −√−1

i4 = +1

i5 = √−1

i6 = −1 ...etc



what is i10?

i10= i4 × i4 × i2

= 1 × 1 × −1

= −1

Let's Express the following in a+ib form:

(5+√3i)/(1-√3i).

Given: (5+√3i)/(1-√3i)

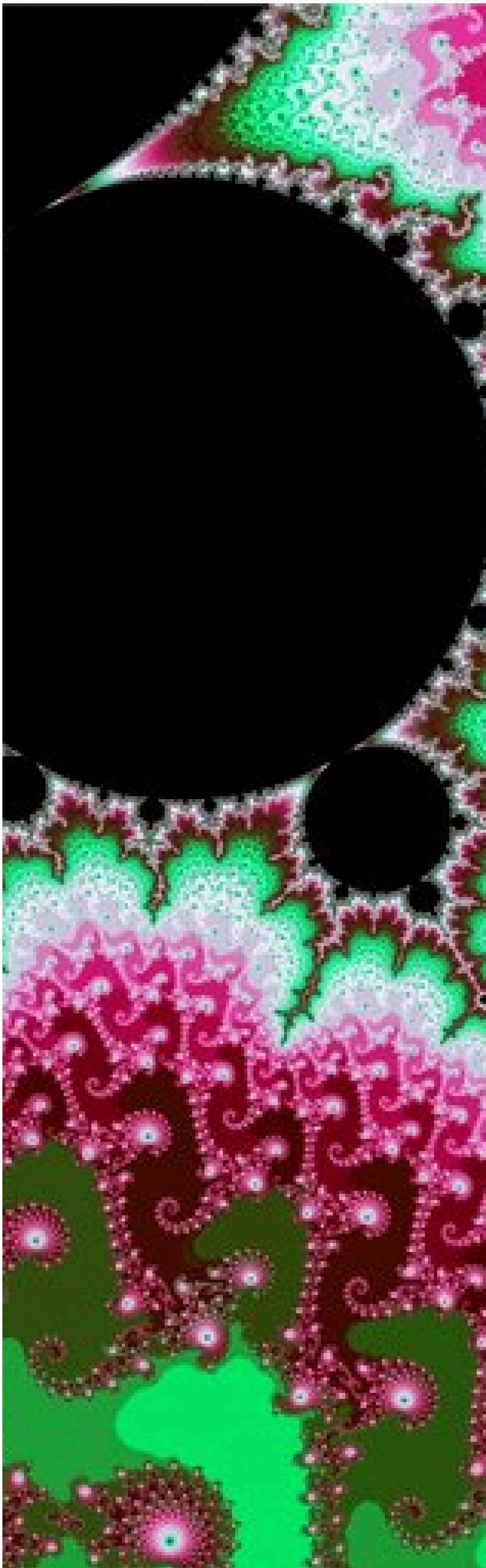
Square root of i has both real and imaginary parts i.e.,

√i = ± √1/2 (1+2)

$$z = \frac{5+i\sqrt{3}}{1-i\sqrt{3}} \times \frac{1+i\sqrt{3}}{1+i\sqrt{3}} = \frac{5-3+6\sqrt{3}i}{1+3} = \frac{1}{2} + \frac{3\sqrt{3}}{2}i$$

$$\text{Modulus, } \bar{z} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{28}{4}} = \sqrt{7}$$

$$\text{Conjugate, } \bar{z} = \frac{1}{2} - \frac{3\sqrt{3}}{2}i$$



Let’s evaluate the square root -121

√(-121) = √(-1 x 121)

= √(-1) x √(121)

= i x (± 11)

= ±11i

Let’s solve x2 + 1 = 0

x2 = −1

x = ± √(−1)

x = ± I

(−i)2 + 1 = (−i)(−i) + 1 = +i2 + 1 = −1 + 1 = 0

+i)2 +1 = (+i)(+i) +1 = +i2 +1 = −1 + 1 = 0

Let’s express the roots of the quadratic equation x2 + x + 1 = 0 in terms of imaginary numbers.

Comparing it with ax2 + bx + c = 0, we get a = 1, b = 1, and c = 1.

Substituting these in the quadratic formula:

x = (-b ± √(b2 - 4ac)) / (4a)

= (-1 ± √(12 - 4 · 1 · 1)) / (4·1)

= (-1 ± √(1 - 4)) / 4

= (-1 ± √(-3)) / 4

= (-1 ± i√3) / 4

The roots are (-1 + i√3) / 4 and (-1 - i√3) / 4.

Resources

https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:imaginary/a/intro-to-the-imaginary-numbers%20https:/www.geeksforgeeks.org/imaginary-numbers/https:/byjus.com/maths/imaginary-numbers/https:/www.mathsisfun.com/numbers/imaginary-numbers.htmlhttps:/www.cuemath.com/numbers/imaginary-numbers/https:/www.math.uri.edu/~merino/spring06/mth562/ShortHistoryComplexNumbers2006.pdfhttp:/matematiktutkusu.comhttp:/khanacademy.comhttp:/Wikipedia.orghttp:/slideplayer.biz http:/steemit.comhttps:/www.google.com/url?sa=i&url=https%3A%2F%2Fmail.karabin.camp%2Fcomputer-art-fractal-abstract-psychedelic-digital-art-image-black-ss-13918189&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABCxAQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.assignmentexpert.com%2Fblog%2Fwhat-are-complex-numbers%2F&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABC5AQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Fmaths.org%2Fstep%2Fstep-2-complex-numbers-0&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDBAQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.coolmath.com%2Falgebra%2F10-complex-numbers%2F01-what-are-complex-numbers-02&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDJAQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Ftwitter.com%2Fqiskit%2Fstatus%2F1532090850058043395&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDRAQhttps://www.google.com/url?sa=i&url=http%3A%2F%2Fwww.mathaholic.com%2F%3Ftag%3Dart&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDZAQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Fcourses.lumenlearning.com%2Fsuny-osalgebratrig%2Fchapter%2Fcomplex-numbers%2F&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDhAQhttps://www.google.com/url?sa=i&url=https%3A%2F%2Fopencurve.info%2Fcomplex-numbers-an-introduction%2F&psig=AOvVaw1c0DQedxRGzhcHd20mSOO-&ust=1714670531439000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqGAoTCPCAl9T77IUDFQAAAAAdAAAAABDpAQ